Transformations — Grade Ten

Ohio Standards Connections

Geometry and Spatial Sense

Benchmark F
Represent and model transformations in a coordinate plane and describe the results.
(Grades 8 - 10)

Indicator 8
Derive coordinate rules for translations, reflections and rotations of geometric figures in the coordinate plane.
(Grade 10)

Indicator 9
Show and describe the results of combinations of translations, reflections and rotations (compositions); e.g., perform compositions and specify the result of a composition as the outcome of a single motion, when applicable.
(Grade 10)

Mathematical Processes Standard

Benchmark C
Recognize and use connections between equivalent representations and related procedures for a mathematical concept; e.g., zero of a function and the *x*-intercept of the graph of the function, apply proportional thinking when measuring, describing functions, and comparing probabilities.
(Grades 8 - 10)

Lesson Summary:
Students will recognize, perform and derive rules for basic transformations including translation, reflection and rotation. They also will consider compositions, a combination of two or more basic transformations. Students will explore both transformations and compositions within a coordinate plane.

Estimated Duration: *100 minutes*

Commentary:

This pre-assessment was designed to be an interactive learning environment for the students guided by the teacher. It is very important that students are not told things during this lesson. Most students should have some experience with basic transformations, but may not be familiar with the terminology. The terminology can be provided at the appropriate time.

It is important to encourage students to observe and then be given opportunities to draw their own conclusions. Allow students to investigate their conclusions, especially if they are incorrect.

Pre-Assessment:

Instruct students that they will have the opportunity to show what they know about basic transformations. They will demonstrate their understanding using two different color "R's".

Each group of four students should be given two student handouts (one of each color). Each student should cut one "R" of each color. The teacher may want to have the student handouts cut in half to simplify the process.

Instruct the students to use one color for the initial position of the "R" and the other color should be placed in the final position of the "R" after the transformation

1. Ask students to perform a translation using the two "R's" at their desks. Give students time to try this on their own. Select a student to describe what they did for the class.

   Select a student to demonstrate a translation using the two "F" transparency forms and provide an explanation. An alternate strategy is the teacher demonstrating the translation and asking students to provide directions and explanations.

   Ask students to perform a specific translation using the two "R's"; translate about five inches to the right and about one inch down. Circulate around the room to evaluate student understanding and to provide assistance. Assign additional translations, as needed, until most students demonstrate understanding.

2. Ask students to perform a rotation using the two "R's" at their desks. Give students time to try this on their own. Select a student to describe what he/she did for the class.

Other Related Ohio Standards

Geometry and Spatial Sense

Benchmark G
Prove or disprove conjectures and solve problems involving two- and three-dimensional objects represented within a coordinate system.
(Grades 8 - 10)

Select a student to demonstrate a rotation of 90° clockwise about the lower left corner of the "F" and provide an explanation, or perform the demonstration for the class with students providing directions and explanations.

Ask students to use the two "R's" to perform a 180° counterclockwise rotation about the bottom right corner. Circulate the room to evaluate student understanding and to provide assistance. Assign additional rotations, as needed, until most students demonstrate understanding.

3. Ask students to perform a reflection using the two "R's" at their desks. Give students time to try this on their own. Select a student to describe what he/she did for the class.

   Select a student to use the two "F" transparency forms to demonstrate a reflection over the top edge and provide an explanation, or perform the demonstration for the class with students providing directions and explanations.

   Ask the students to use the two "R's" to perform a reflection over the left edge. Circulate the room to evaluate student understanding and to provide assistance. Assign additional reflections, as needed, until most students demonstrate understanding.

Scoring Guidelines:
Scoring should be informal. Observe and provide assistance as needed. Students also should be encouraged to help one another if they cannot perform the specific transformations.

Post-Assessment:

- The students will provide in their own words a description of each of the basic transformations (i.e., translation, rotation and reflection). In the description they should compare and contrast the transformations and discuss how compositions (i.e., combinations of basic transformations) can be used to yield a single transformation (e.g., two reflections can yield the same result as a single translation).
- The students will write a description of how parallelogram STUV was transformed to create parallelogram S’T’U’V’. In the writing, the student should use appropriate mathematics terminology, include descriptions of the intermediate locations of the parallelogram, and provide sufficient explanation to demonstrate their understanding of composition.

  Information students need for this task:

  STUV is defined by points S(-6, 2), T(-1, 2), U(-3, 5) and V(-8, 5) and S’T’U’V’ is defined by points S'(6, -2), T'(1, -2), U'(3, -5), V'(8, -5).

Scoring Guidelines:
4 = Describes the basic transformations and composition correctly, demonstrates understanding of how to perform the basic transformations, uses the mathematical terminology correctly, includes a diagram on graph paper that is clearly labeled.

3 = Does not describe the basic transformations and composition correctly, but

demonstrates understanding of how to perform the basic transformations, uses the mathematical terminology correctly for most things, includes a diagram on graph paper that is clearly labeled.

2 = Describes the basic transformations and composition correctly or demonstrates understanding of how to perform the basic transformations, does not use the mathematical terminology correctly, does Not include a diagram on graph paper that is clearly labeled.

1 = Describes the basic transformations, but does not describe composition correctly or does not demonstrate understanding of how to perform the basic transformations, or does not use the mathematical terminology correctly, or does not include a diagram on graph paper that is clearly labeled.

0 = Does not attempt or no evidence is shown.

Instructional Procedures:

1. Distribute the materials. Instruct students to put the transparency on top of the graph paper so that the edges align and then draw and label the *x*-axes and *y*-axes axes on the transparency. Instruct students to always align axes before drawing and labeling the initial position for triangles.
2. Instruct students to draw a triangle, Δ ABC, in the first quadrant and label the vertices. Use the translation table on the activity worksheet (Blackline Master #1) to record the points of the vertices in the initial location row.

> **Instructional Tip:**
> Encourage students to draw different types of triangles such as, obtuse, right or acute. Steer some students away from using equilateral and isosceles triangles. Additionally, encourage students to draw their initial triangles in quadrants other than the first quadrant.
>
> If any of these adjustments are made, be sure to engage the students to explore comparisons during the discussions. For example, students may: compare the results for different types of triangles; be asked to make conjectures about if the type of triangle used for the transformation will affect the rule; or consider if the rules work if the original triangle is in a different quadrant.

3. Instruct students to perform a translation of 3 right and 2 up by moving the transparency appropriately over the grid paper (Note: after the move the edges of the graph paper and the transparency should no longer be aligned). Then record the vertices of Δ ABC in the Translation 1 row of the table.
4. Invite students to compare the coordinates of the Initial Location and the Translation 1 rows and then write a rule that will hold for every translation.

   If they cannot write a rule, encourage them to try another translation that they choose and examine the coordinates for a pattern. (Remind students to realign the transparency and the graph paper before performing the next

translation.) Then, write the rule.

Give students ample time to investigate the translations. If students continue to struggle after trying multiple translations independently, encourage them to partner with a neighbor for collaboration.

Teacher Tip:
For each of the transformation exercises, be sure to let students struggle if the answers are not immediately forthcoming. The best way to help is to give hints through questioning, but try not to tell students how to proceed or the answer.

Sample questions might include:

- Do you see a pattern among the *x*-coordinate values?
- How are the *y*-coordinate values changing?
- Are any of the values remaining constant?
- What observations can you make about the translation directions (3 right and 2 up) and the coordinates?

After students have written a rule, encourage them to create another translation (or two) to test their rule. Remind them to use both positive and negative numbers for their tests.

5. Students should clean their transparencies (ensure that the axes are drawn, labeled, and aligned with the graph paper axes). Then start with a new triangle, $\triangle ABC$, drawn in the first quadrant. Record the coordinates of the vertices in the table for Rotation in the Initial Location 1 row.

6. Instruct students to perform a rotation of 90° clockwise about the origin. To do this, students need to turn their transparencies one quadrant clockwise and align the origin and axes (*x* to *y* and *y* to *x*). It is not advised that students be given this information, but provide hints if they are confused or have trouble getting started. If they request a protractor, loan them one. After the rotation is completed, record the coordinates of the vertices in the 90° clockwise row of the table.

7. Invite students to compare the coordinates of the initial locations and the 90° clockwise rows and then write a rule that will hold for every 90° rotation starting from the first quadrant.

   If they cannot write a rule with the information they have, encourage them to try another 90° rotation using a different triangle, then re-examine the coordinates for a pattern. (Remind students to realign the transparency and the graph paper before drawing the new triangle and performing the next rotation.) Then write the rule.

   Give the students ample time to investigate rotations. If students continue to struggle after trying multiple rotations independently, encourage them to partner with a neighbor for collaboration.

   After they have written a rule, encourage them to try other 90° clockwise

rotations using different triangles (obtuse, right, etc.) in the first quadrant or other quadrants (if they are ready) to test their rule.

8. Students should clean their transparencies (ensure that the axes are drawn, labeled, and aligned with the graph paper axes). Then start with a new triangle, $\triangle$ ABC, drawn in the first quadrant. Record the coordinates of the vertices in the table for Reflection in the Initial Location 1 row.

9. Instruct students to perform a reflection over the *y*-axis. To do this, students need to flip their transparencies over and align the origin and axes (*x* to *x* and *y* to *y*). It is not advised that students be given this information, but provide hints if they are confused or have trouble getting started. After the reflection is completed record the coordinates of the vertices in the Reflection over *y*-axis row of the table.

10. Students should compare the coordinates of the initial location and the reflection over *y*-axis rows and then write a rule that will hold for every reflection over the *y*-axis.

    If they cannot write a rule with the information they have, encourage them to try another reflection over the *y*-axis using a different triangle, then re-examine the coordinates for a pattern. (Remind students to clean and realign the transparency and the graph paper before drawing the new triangle and performing the next reflection.) Then write the rule.

    Give the students ample time to investigate reflections. If students continue to struggle after trying multiple reflections independently, encourage them to partner with a neighbor for collaboration.

    After they have written a rule, encourage them to try other reflections over the *y*-axis using different triangles in different quadrants to test their rule.

11. Instruct students who finish early to answer the Challenge and Summary questions on the worksheet (Blackline Master #1). For those who do not have time to complete the questions during class should complete them with their homework assignment.

12. Facilitate a whole class discussion about the student's discoveries about the basic transformations. Record key student discoveries for the class (i.e., on chalk board or overhead). Invite students to share or to demonstrate work that might add to the discussion. Be sure to discuss the challenge questions. Leave the summary questions for individual reflection.

13. Pose the following problem: Given trapezoid ABCD, where A(-11, 4), B(-13, 7), C(-4, 7) and D(-6, 4). What basic transformations can be used to get trapezoid WXYZ, where W(-1, 3), X(2, 5), Y(2, -4) and Z(-1, -2)?

    Organize students in pairs to provide a response. If students have trouble getting started, encourage them to use their transparencies and graph paper for demonstration. Allow 5 to 15 minutes for student work.

14. Ask for volunteers to share their findings. There are multiple solutions for the problem. The key that you are looking for is that it takes a combination of two or more basic transformations to go from trapezoid ABCD to get to

trapezoid WXYZ. Record responses from several students, then have students individually select and verify two or three of those recorded.

15. Before ending the discussion introduce students to the term composition (the combination of two or more basic transformations). If time remains have students write their understanding about compositions and what they might be used for in everyday life or the workplace. Examples appropriate for most grade 10 students might include interactive games that involve placing colored shapes into spaces or packing luggage into the trunk of a car.

Differentiated Instructional Support:
Instruction is differentiated according to learner needs, to help all learners either meet the intent of the specified indicator(s) or, if the indicator is already met, to advance beyond the specified indicator(s).

- Review how to plot points on a graph.
- Encourage peer tutoring throughout the lesson.
- Use matrices to model basic transformations.

Extension:
These are ideas for all students to continue learning on this topic - in the classroom or outside of the classroom.

Students should perform more advanced transformations such as the following:

- 180° rotation and derive a rule.
- 90° counterclockwise rotation and derive a rule.
- Reflection over the line $y = x$ and derive a rule.
- Perform a transformation in a coordinate plane and then use coordinate geometry to prove that the resulting shape is congruent to the original shape.

Homework Options and Home Connections:

1. Draw a pentagon on the grid transparency with the following points: A(2,-4), B(6,-4), C(8,-7), D(5,-8), E(0,-6)

2. Find the coordinates of the vertices of the pentagon after performing each transformation
   - Translation of 5 to the left and up 3.
     Answer: A(-3,-1), B(1,-1), C(3,-4), D(0,-5), E(-5,-3)
   - Rotation of 90° counterclockwise.
     Answer: A(4,2), B(4,6), C(7,8), D(8,5), E(6,0)
   - Reflection over the *x*-axis.
     Answer: A(2,4), B(6,4), C(8,7), D(5,8), E(0,6)

Or, assign appropriate problems from your curriculum materials.

Key Vocabulary:

- translation
- rotation
- reflection
- composition

Technology Connections:

- Geometry computer software or calculator with geometry software may be used to model transformations and compositions.

Materials/Resources Needed:

For the teacher: Overhead projector, screen, Letter R Master (Blackline Master #2), Letter F Master (Blackline Master #3), grid paper transparency, water soluble marker(s). Sufficient copies of the Activity Worksheet (Blackline Master #1) using two colors so that each student has one "R" of each color. Protractors should be available in case students request them.

For the students: Two "R" shapes on two different colors of paper, coordinate graph paper with *x*-axes and *y*- axes drawn, ruler, one overhead transparency, overhead markers, moist paper towels (to erase film), scissors to cut out "R" shapes (not needed if teacher pre-cuts letters), and activity worksheet.

Attachments:

- Activity Worksheet (Blackline Master #1)
- Letter R Master (Blackline Master #2)
- Letter F Master (Blackline Master #3)
- Commentary on Student Work
- Sample Student Work A
- Sample Student Work B