



# Using Representations to Model Problems — Grade Five

## **Ohio Standards Connections**

### **Patterns, Functions and Algebra**

#### **Benchmark B**

Represent, analyze and generalize a variety of patterns and functions with tables, graphs, words and symbolic rules.

(Grades 5 - 7)

#### **Indicator 3**

Use variables as unknown quantities in general rules when describing patterns and other relationships.

(Grade 5)

#### **Benchmark F**

Use representations, such as tables, graphs and equations, to model situations and to solve problems, especially those that involve linear relationships.

(Grades 5 - 7)

#### **Indicator 5**

Model problems with physical materials and visual representations, and use models, graphs and tables to draw conclusions and make predictions.

(Grade 5)

### **Mathematical Processes Standard**

#### **Benchmark H**

Use representations to organize and communicate mathematical thinking and problem solutions.

(Grades 5 - 7)

#### **Benchmark J**

Communicate mathematical thinking to others and analyze the mathematical thinking and strategies of others.

(Grades 5 - 7)

#### **Benchmark K**

Recognize and use mathematical language and symbols when reading, writing and conversing with others.

(Grades 5 - 7)

### **Lesson Summary:**

*The student will make predictions or draw conclusions based on physical and visual representations of profit and sales*

**Estimated Duration:** 120 minutes

### **Commentary:**

This lesson is not designed as an introduction to using graphs to represent data. Students need to have an understanding of how to make graphs prior to this lesson.

### **Pre-Assessment:**

The pre-assessment is designed to assess understanding and skills in using a table of values to create a graph. The assessment also will provide information related to students' skill in generalizing data from either the graph or the table and to what level are they able to generalize. Are students able to verbalize the rule or can they take it to the next step and write an expression for the rule?

- Give all students a table of values. The table should represent a linear relationship.
- Ask the students to graph the set of points on a coordinate system.
- Ask the students to generalize the data in the graph by writing a rule.

### **Scoring Guidelines:**

A teacher/class developed rubric should be used to evaluate student responses.

Criteria should include:

- Graph is labeled correctly.
- The domain and range of the graph is correct for the data.
- The points are plotted correctly.
- A general rule or statement about the graph is provided.

### **Post-Assessment:**

Use the Post-Assessment worksheet provided with the lesson.

This assessment has each student representing the sale of an additional item with a table, graph and expression.

### **Scoring Guidelines:**

Student work should be checked for understanding using the answer sheet provided.

### **Commentary:**

When scoring the post-assessment, consider all responses based on evidence of understanding of the concept. For example, when looking at the graph, if the student incorrectly determined the profit but has correctly completed all other parts of the graph based on the incorrect profit, then intervention should be provided to the student to build understanding of profit. Further evaluation may be required to determine if the error in finding the profit is a calculation error and not a misunderstanding of profit.

# Using Representations to Model Problems — Grade Five

## Other Related Ohio Standards

### **Patterns, Functions and Algebra**

#### Benchmark E

Use rules and variables to describe patterns, functions and other relationships.

(Grades 5 - 7)

#### Benchmark K

Graph linear equations and inequalities.

(Grades 5 - 7)

#### Benchmark L

Analyze functional relationships, and explain how a change in one quantity results in a change in the other.

(Grades 5 - 7)

#### Indicator 6

Describe how the quantitative change in a variable affects the value of a related variable; e.g., describe how the rate of growth varies over time, based upon data in a table or graph.

(Grade 5)

### **Data Analysis and Probability**

#### Benchmark J

Compare experimental and theoretical results for a variety of simple experiments.

(Grades 5 - 7)

#### Indicator 10

Compare what should happen (theoretical/expected results) with what did happen (experimental/actual results) in a simple experiment.

(Grade 5)

#### Benchmark K

Make and justify predictions based on experimental and theoretical probabilities.

(Grades 5 - 7)

#### Indicator 11

Make predictions based on experimental and theoretical probabilities.

(Grade 5)

Likewise, additional assessment may be required to determine if errors in other responses represent a misunderstanding of the concepts, an error in set up of the table or graph, or difficulty in making a generalization.

## **Instructional Procedures:**

1. Organize students into small groups.
2. Engage the class in the following scenario:
  - In order to raise funds to subsidize a trip to the state park, each student will participate in a fundraiser. The goal is for each student to raise \$5.00. The student council is in the process of selecting an item to sell during the fundraiser.
  - The task of this class is to make recommendations to the student council based on the analysis of the sale items.
3. Student groups will randomly select an item from the table to analyze. (Put each item on a slip of paper and have each group randomly draw one from a hat or jar.)

### **Teacher Tip:**

This is a good time to review the topic of "randomness" and probability. Have the students determine the probability of selecting any specific item.

Item	Price	School Cost
Pencil	\$.15	\$.10
Pen	\$.25	\$.15
Pencil Top Eraser	\$.25	\$.10
Small Bounce Ball	\$.40	\$.25
Licorice	\$.50	\$.25
Pencil Pouch	\$.75	\$.35
Candy Bar	\$1.00	\$.60

4. Students will determine the amount of "profit" that their item will generate each time an item is sold.

### **Commentary:**

Some students may not have an understanding of the term profit. This concept may have to be discussed with them at this time. In this situation, profit is the difference in school cost and the selling price of the item. This is the amount of money that will be used to help pay for the trip to the state park.

## Using Representations to Model Problems — Grade Five

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5. Each group will be required to illustrate the amount of profit generated by the sales of the item selected through two different representations.
- The first will be a table of values to represent the possible sales of the item.
  - The second representation will be plotting the points from the table on a graph.

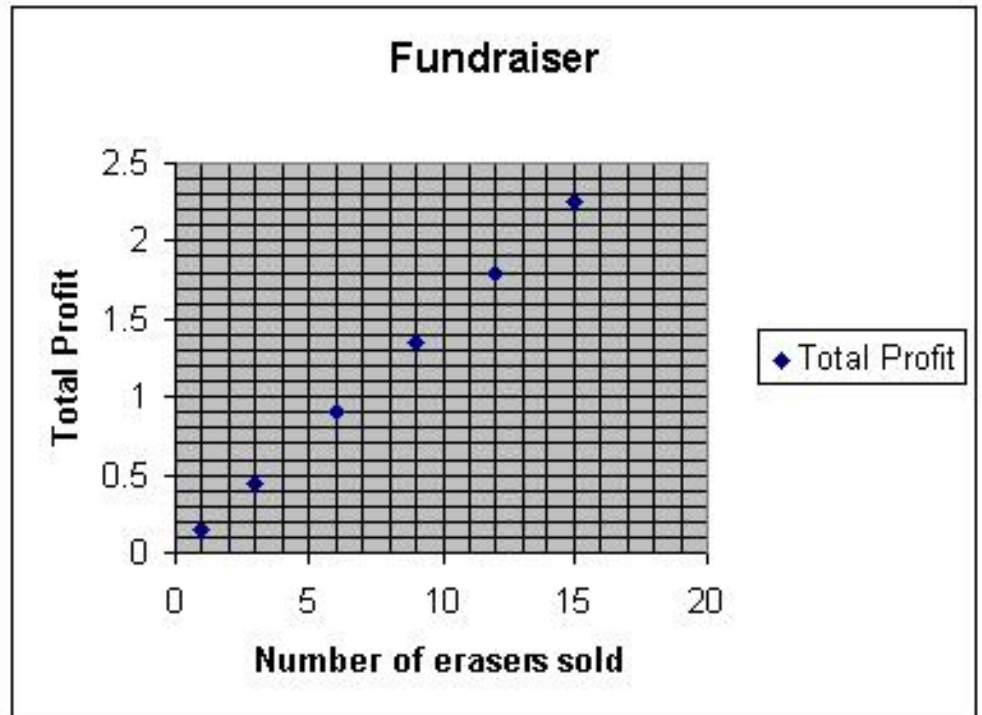
**Teacher Tip:**

Discuss with the students the labeling and scale that needs to be used with each of the representations.

A sample table and graph are provided to illustrate the expectations.

Number of erasers sold	Total Profit
1	\$.15
3	\$.45
6	\$.90
9	\$1.35
12	\$1.80
15	\$2.25

## Using Representations to Model Problems — Grade Five



**Teacher Tip:**

As with any project it is advantageous for the student to create a draft of the representation before completing the final document. This will allow the teacher to give input on the student's work before it is completed.

**Commentary:**

A misconception that some students have is that the numbers chosen have some significant meaning. For example, if multiples of 5 are used for the number of items sold in an initial table then students tend to always want to use multiples of 5. Therefore, when creating the table have groups use different values to represent the number of items sold.

6. Use the tables and graphs created in the groups. Have the students make conjectures about the relationship of sales to amount of profit. Instruct each group to generalize their data with a mathematical sentence (rule) or a narrative that describes the relationship between sales and profit.

**Teacher Tip:**

Possible conjectures could include: the points on the graph are in a straight line or the amount that it increases is always the same. Possible rules may include: multiply the number of items sold by the profit, multiply \$.35 times the number of items sold, or  $\$0.35x = T$ . If a student response is  $\$0.35x = T$ , further questioning should follow to determine what the variables  $x$  and  $T$  represents.

## Using Representations to Model Problems — Grade Five

7. Instruct each group to:
  - Determine how many items one person will need to sell to make the goal of \$5.00.

**Commentary:**

This may be done in several ways, depending on the item used by the group. For example, some graphs may extend far enough for students to approximate how many items are needed to meet the goal. While other students may determine the number needed by finding the amount required to equal \$1.00 in profit and then multiplying that value by 5. The focus is to use the information at hand to make a prediction.

- Determine how many items the group would need to sell to make the goal for the group.
  - Determine how many items the group would need to sell to make the goal for the class.
8. Direct the students to transfer their representations to chart paper or graph paper.
  9. Display all representations for students to see.
  10. Have each group compare and contrast their representations with representations of other groups, discussing how they are alike or different. (Look at the scales used by different groups and discuss how the different sets of data would look on the same scale.)
  11. Direct the students to discuss as a class the following:
    - Does it make sense to connect the points to create a line graph? Why or why not?

**Teacher Tip:**

This is a good time to discuss the notion of independent points to represent the data. However, drawing a line through the point could be used to generalize the data and making predictions.

- What conversations took place while creating the representations?
  - Do all graphs convey the same message? What aspects of the graphs are the same and what are different?
  - Which representation is easier to interpret (table or graph)? Why?
  - Which representation (table or graph) best depicts the situation at hand? Why?
  - Is the corresponding expression or rule correct?
  - Which group will reach their goal first? Why?
12. Instruct each student to create a report to present to the student council supporting one of the items to be used for the school fundraiser. The student should support their reasoning for selecting the item by referencing the tables

## Using Representations to Model Problems — Grade Five

and graphs created by the class.

### **Differentiated Instructional Support:**

Instruction is differentiated according to learner needs, to help all learners either meet the intent of the specified indicator(s) or, if the indicator is already met, to advance beyond the specified indicator(s).

- Work with a ten-cent profit for easier computation and depiction.
- Allow students who have difficulty using decimal notation to use 35¢ rather than \$.35.
- Have students investigate how the data and representations change as the price changes. What would happen to the representation if the price increases or if the price decreases? How does this change the rule or expression?
- A graphing calculator or computer software can be used to generate or evaluate graphs produced by the students.

### **Extension:**

This extension takes the lesson in a new direction by having the students investigate how the situation given in the lesson may play out in the real world. Students are investigating the effects of sales on the total amount of money acquired over time.

- Re-organize each group using a jigsaw method, having one member from each of the original groups form a new group.
- Have each student in the group simulate the sale of their item. Students who were selling the candy bar or pencil pouch will use one number cube. Two number cubes will be used by the students who were selling licorice and small bounce ball, while those who were selling pencils, pens and pencil top erasers will use three number cubes.
- Instruct students to simulate two days of sales. Record this data in a table and create a graph. Using the graph and table, predict how many days it may take to meet their goal of \$5.00. Have the students write an expression to represent their simulation.

### **Teacher Tip:**

Have the students continue the simulation and use additional data to verify predictions based on only two data points. This can also lead to a discussion about why a better prediction can be made with more data.

- Have members of the group share their tables, graphs and predictions with the other members of their group. Have students discuss their results.
- Facilitate a discussion with the whole class using questions such as:
  - Compared to the original prediction, which group would meet their goal first? Did the simulation come up with the same outcome?
  - Why did different groups use a different number of number cubes during the simulation? (Related to supply and demand as well as the cost; the higher the cost the fewer items will be sold.)
  - Instruct the students to re-evaluate their presentation to the student council based on the data collected during the simulation. Students may determine that a different item should be used for the school fundraiser.

# Using Representations to Model Problems — Grade Five

## **Homework Options and Home Connections:**

Have the student create a graph to represent the money that they save or spend over the next week.

## **Interdisciplinary Connections:**

Connections could be made with this lesson and the social studies standards for grade five.

Social Studies

Economics

Benchmark C. Explain how competition affects producers and consumers in a market economy and why specialization facilitates trade.

Indicator 7. Explain why competition among consumers/buyers results in higher product prices.

## **Materials/Resources Needed:**

*For the teacher:* Hat/container, papers with names of items to be sold.

*For the students:* Chart paper, grid paper, markers, paper and pencil, number cubes.

## **Key Vocabulary:**

- profit
- conjecture
- cost
- representation
- subsidize

## **Technology Connections:**

Using a graphing calculator or graphing programs to create the representations for this lesson can help students construct dynamic, visual models of the relationships that exist between sales and profit.

- Students can see their data represented in multiple formats (e.g., table, plotted points on a graph) that are linked. Changes to the data set are immediately reflected in all graph formats. For example, when students add a new piece of data to their table (e.g., 3 erasers sold, \$.45 profit) that data point will automatically appear on their graph (3, .45). Moving a point on their graph will result in parallel changes to the corresponding values listed in their table. Different graph formats can be placed side-by-side on the screen, helping students to more clearly see the kinds of relationships that exist between different representations and the ways that each piece of added data impacts their graphs.

## **Commentary:**

As an extension to the lesson above, students can view their data in additional graph formats. Students can continue to make the same types of comparisons between these representations as described in the lesson above. Which representation is easier to interpret and why? Which representation best depicts the situation at hand? Why?

## Using Representations to Model Problems — Grade Five

- Graphing programs enable students to easily create and edit graphs. Students can quickly plot data points and interpret the resulting patterns. This frees students to experiment with their graphs in order to address questions such as those listed above. What would happen to their graph if the price increased? How would their graph change if the price decreased?

**Attachments:**

- Post Assessment Worksheet
- Post Assessment Answer Sheet