Number Subsets: Winning the Number Game — Grade Eight

Ohio Standards Connections

Number, Number Sense and Operations

<u>Benchmark B</u>
Identify subsets of the real number system.
(Grades 8 - 10)

<u>Indicator 2</u>
Recognize that natural numbers, whole numbers, integers, rational numbers and irrational numbers are subsets of the real number system.
(Grade 8)

Mathematical Processes Standard

<u>Benchmark A</u>
Formulate a problem or mathematical model in response to a specific need or situation, determine information required to solve the problem, choose method for obtaining this information, and set limits for acceptable solution.
(Grades 8 - 10)

<u>Benchmark B</u>
Apply mathematical knowledge and skills routinely in other content areas and practical situations.
(Grades 8 - 10)

Lesson Summary:
Students will review their understanding about numbers and number systems through playing the Number Game. Students will create a list of numbers and based upon the characteristics and properties of the numbers they will earn points. Students will initially play the game using scoring criteria set by the teacher. Then students will have an opportunity to set the scoring criteria and compete against the teacher and their peers.

Estimated Duration: *50 minutes*

Commentary:
This lesson design can be used multiple times throughout the year with eighth-grade students as well as with students in other grades. This flexible design is easily adapted to fit many mathematical concepts and is highly engaging for the students. The approach of the lesson is particularly well suited for those concepts that have specific characteristics or properties associated with them, like number types or geometric shapes.

This lesson was pilot-tested by teachers across the state of Ohio. Some of the teacher comments about this lesson follow:
- *"This is a fun, engaging activity that can be used at all times."*
- *"I love the student involvement!...Having students generate the definitions in lieu of the teacher…"*
- *"It was easy to do yet accomplished the goal. It was a fun activity for students. It could be expanded upon in many ways to meet the needs of a diverse group of kids."*
- *"Thanks for sharing this great lesson with me and my students!"*

Pre-Assessment:
The goal of this assessment is to determine how much students know about the real number system.
- Ask students to write a set of their 10 favorite numbers.
- Ask students to provide names (or labels) to represent the type of number for each number listed. Instruct students that they should use as many different names as they can for each number on their list.
- Circulate the room and look for names such as integers, whole numbers, rational (or fraction), irrational, prime, etc. If students appear to be stuck, provide a hint like, "Does anyone have any whole numbers? Can anyone name other types of numbers?" Do not provide answers. Just encourage students to try.
- Invite students to share some of the names they used. Facilitate a discussion about the specific number types until most students have some familiarity with the number types you plan to use during the game.

Scoring Guidelines:
This activity is used to give the teacher an idea about what students know about the mathematical concepts that will be targeted during the game, for this specific

lesson the concept is number types.

This assessment was not designed to generate a grade. This should be evaluated informally by walking around the room and examining the types of names the students use for their numbers. Take notes about any numbers that have been mislabeled so that those types of numbers can be included during the discussion or encountered during the game.

Post-Assessment:
- Students will create their own versions of the number game by creating rules for play (if needed) and the criteria for scoring the game. The rules and/or scoring criteria should be written clearly and be easily understood.
- Set the specific number of scoring criteria students will need to create based upon lesson objectives and the depth of the concepts targeted. For example, the scoring criteria must include at least four different number types that are subsets of the real number system. For added clarity, the student should provide examples and non-examples of the number types used (all examples must be different from those used during the class).
- Students must create a sample list of numbers that would earn a high score using the criteria and based on the concepts used. For this lesson the sample list would be a set of 10 numbers.
- Students must be sure to label numbers correctly with all of the number types that apply (e.g., -3 is an integer, rational number and is negative).

Teacher Tip:
Suggest that students play their games multiple times to be sure that they have generated a **good** set of sample responses. You may want students to write their criteria on index cards to make them easier to store for future use.

Scoring Guidelines:
A sample scoring rubric is provided below. It is a good idea to have students help generate the rubric at the time of assignment. The students would benefit by getting a better understanding of what is expected of them.

4 = Clear guidelines written for the game's scoring criteria; examples and non-examples were included; the sample list of 10 numbers were labeled correctly with all of the number types; and the numbers represented a good set for winning the game given the scoring criteria.

3 = Clear guidelines written for the game's scoring criteria; and examples and non-examples were included for **_some_** of the number types included; or, the sample list did not represent a **_good_** set of numbers for winning the game given the scoring criteria; e.g., the resulting score is relatively high; and, the sample list of 10 numbers were labeled correctly with some of the number types.

2 = Guidelines written for the game's scoring criteria lack clarity; examples and non-examples were omitted; and the sample list of 10 numbers did not represent a **_good_** set for winning the game given the scoring criteria; and, the sample list of 10 numbers were labeled correctly with **_some_** of the number types; or, **_some_** of the numbers were labeled incorrectly or labels were missing.

1 = Guidelines written for the game's scoring criteria lack clarity; and examples and non-examples were omitted; and the sample list of 10 numbers were omitted.

0 = No attempt was made or no evidence of understanding displayed.

> **Teacher Tip:**
> *The following comment was made by one of the teachers who participated in the pilot test:*
>
> *"It was difficult for me, at first, not to blurt out answers to the students' questions like, 'Is 89 prime?' or 'What is an irrational number?'. But, the more we played, the better we all became. When students realized that I was not going to answer direct questions, they referred to the chart to answer their own queries. They seemed to enjoy my going around the room and complimenting them on number choices or my saying, 'There's an integer!' or 'Tom has a composite number!' It was a lesson filled with great interactions!"*

Instructional Procedures:

1. Direct students to focus on their list of 10 numbers from the pre-assessment activity or they may generate a new list. Decide and instruct accordingly.

2. Instruct the students for playing the Number Game as follows:
    *   Points are earned according to the list of numbers and the scoring criteria.
    *   Questions may be asked at any time during the game. Inform students that unasked questions may result in lost points.
    *   The game will be played more than once. Inform students that the more times they play, the better their scores will get.
    *   Prizes may be awarded (optional) based upon who has the highest score. Adjust this according to your preference and goals (e.g., top five scores, everyone who participates, etc.).

> **Teacher Tip:**
> *Another teacher in the pilot test offered the following comments: "I believe the main tip is to remember that the students will not do well at first, but the learning curve with this lesson is quite steep. You must have patience and 'bite your tongue'.... The assessment activity is a good one, where students create their own criteria and then have the rest of the class play the game using their rules. The students were quite proud of their work, and proud that we used their rules and not just the teacher's."*

3. Create criteria based upon observations of students' understandings from the pre-assessment. For example, if no one labeled numbers as rational, then be sure to award significant points for rational numbers.

4. Here are sample criteria that could be used to play the game based on number types for the first game:
    *   2 points for each integer in your list.

- 2 points if the product of all numbers in your list is negative.
- 5 points if the sum of the natural numbers is greater than 500
- 9 points if the smallest whole number is included in your list.
- 7 points for each prime number.
- 2 point for each rational number.
- 1 point for each number represented as a decimal.
- 1 point for each number represented as a fraction.
- 10 points for each irrational number (or alternately each real number that is not rational).

Note: Encourage students to use numbers multiple times when applying scoring criteria when numbers belong to different number types. For example, -3 on their list could earn points for being an integer, negative and rational.

Commentary:
Hints for managing the learning during the game follow:
- *Adjust the criteria to target the specific number types identified during your observations during the pre-assessment.*
- *Play using the same set of criteria at least three times. Students should select new numbers for each round to increase their score from the round before. This also enables them the opportunity to classify more numbers.*
- *Pause sufficiently before moving on to the next criteria and observe student scoring looking for mistakes or lack of understanding.*
- *Expect students to ask questions. Do not assume that students are familiar with the number types, even if you have covered them.*
- *If they do not ask questions, try to elicit questions from them by saying things like, "Is everyone able to score using this criteria?" or ask for someone to share.*
- *Do **not** answer questions directly. Instead ask students for examples and non-examples. Using these examples, try to lead the students to a reasonable definition or description for the number type.*

  Teacher Tip:
  The student's definitions must be correct, but do not have to be formal. So, prior to the lesson, be sure to brush up on your understanding of the number types or other definitions you expect to encounter during the game. This will prepare you to make better decisions about students' informal definitions and their validity.

- *Record all number type examples and non-examples for students to see. You may want to make a table to display the number type with its related examples and non-examples.*
- *If students have difficulty generating examples or non-examples, help them by pointing at examples on student papers, relating the*

> *new number type to one that they know, or by asking leading questions.*
>
> **Teacher Tip:**
> Try not to erase examples when going from one number type to another. Organize examples and non-examples according to the space available.
>
> - *Provide the criteria one at a time using media that enables all students to see criteria in written format (e.g., overhead, white board, Power Point presentation, etc.).*
> - *Award prizes appropriately to encourage participation and effort.*

5. Organize students in pairs. Give student pairs time to create their own game scoring criteria. Use instructions very similar to those you plan to provide for the post-assessment.

6. Combine pairs to form groups of four to six students and instruct groups to play each of the newly created games. Let students compete against their peers playing their versions of the game for a set amount of time. Students should provide feedback to their peers about their game especially focusing on the clarity of their criteria (e.g., I do not understand what type of number you want me to find in order to earn three points. What is a natural fraction? etc.).

> **Commentary:**
> *One of the teachers who participated in the pilot asked students if they liked the game, why or why not. Several student responses to that question follow:*
> - *"I thought this activity was fun. I also thought it was hard too because you really have to think about it. I think we should do games like this more often. This game helped us with all the other names for a number, for example rational, whole, etc."*
> - *"Yes, I did like this game because we received rewards for high scores. This game was also fun for a change. This was also a hands-on activity and I do like hands-on activities. They should make more fun games like this. It would make learning a whole lot more fun and more interesting instead of book work."*
> - *"No, I did not like this activity because I didn't have fun playing it. It didn't really teach me more about the integers and rational numbers, etc. than I already knew. Also, it wasn't very grabbing so you wanted to play more, for me at least. That's why I didn't enjoy this math activity."*
> - *"I like this activity. We got to create our own way. It was cool!"*
> - *"I liked this activity because it was educational. It was fun to play. While we were having fun we were still learning about the different kinds of numbers. We should do this more often."*
> - *"I liked this activity because we got to use our imagination and try to figure out what criteria would make the answer over 100."*

> • *"I like this activity because it helps me review my types of numbers. It helps me remember the types of numbers there are."*

Differentiated Instructional Support:

Instruction is differentiated according to learner needs, to help all learners either meet the intent of the specified indicator(s) or, if the indicator is already met, to advance beyond the specified indicator(s).

- Before encouraging the whole class to generate definitions or descriptions for a specific number type using the examples and non-example, partner students to discuss similarities and differences. This will enable more think time and the opportunity to enable peer tutoring.
- Provide time for students to write about their understanding of each number type discussed during the day.
- Change the goal of the game, but keep the criteria (i.e., one game the goal is to get the highest score, the next game the goal is to get the lowest score).
- Introduce negative numbers into the scoring criteria (e.g., subtract -3 if you have three integers).
- Include conditional scoring (e.g., if your list has at least one negative number then square the smallest positive integer and add that value to your score).

Extension:

These are ideas for all students to continue learning on this topic - in the classroom or outside of the classroom.

- Challenge students to create a set of numbers that will score high no matter what criteria are used (the underlying assumption is that all criteria include positive score points and the goal of the game is to score high).
- Let the student groups select Number Games from their group that were fun or engaging to share with the class. Set the extra games aside to play on days when extra time is available. This will enable students to revisit the number types as the year progresses.

Homework Options and Home Connections:

- Students locate the definitions of selected number types using their mathematics textbook and other sources and then compare those definitions to the descriptions generated in class. Next, students write a comparison between what they wrote and the definitions found in the resources. If the textbook definitions are unclear, students write questions to seek clarity.
- Students select a partner to trade game scoring criteria and lists. Students then verify the score for the given list and then create a new list to beat the game creator's score. The students may play one or both games multiple times to determine who can get the highest score. Create a high score list to post the names of students with top scores for the class. (Top scores must be verified by both partners.) Award prizes accordingly.
- The post-assessment may be assigned as homework.

Key Vocabulary:

- natural numbers

- counting numbers
- whole numbers
- rational numbers
- integers
- irrational numbers
- prime numbers
- composite numbers
- negative number
- positivenumber
- real numbers

Teacher Tip:
A teacher in the pilot test offered the following comments: "The dialogue between the teacher and students playing the game is very useful. It clarifies how the game is to be played, even though the directions are very clear. Great activity and format is friendly."

Commentary:
Dialogue between a teacher and students while playing the Number Game

Ms. Black surveys the students in her class as she asks them to write down their 10 favorite numbers on a piece of paper. It is the first day of school, and what better way to start a mathematics class than talking about numbers and number systems?

"We can use any numbers?" asks Emilio.

"Any number you wish," replies Ms. Black, as the students begin to think about 10 different numbers. As she inspects the numbers students are writing she sees mostly natural numbers written on the students' papers. The first time they play the Number Game, the students are thinking inside of the box and writing down numbers they deal with on a daily basis. Ms. Black knows that after one encounter with the scoring criteria, the students will have 10 new favorite numbers.

"Okay, now that everyone has written 10 numbers on their paper, it is time to play the Number Game. Give yourself points according to the scoring criteria that I will share with you. If you have questions, please ask," she says as she begins to reveal the first criteria on the overhead which reads, "two points for every even integer."

"What is an integer?" Amy asks from the middle of the room.

Ms. Black was expecting this question, and she immediately asks the class a different question. "Does anyone know what an integer is or can give an example of one?" she asks. When no one responds, she walks around the room and points at different students' papers. "There's one. There's another one. Wow! You have integers all over your papers. Can someone provide an example of one now?"

Chris raises his hand and says, "three." A few more students provide numbers,

and Ms. Black writes them on the chalkboard. She includes a couple of negative integers in the list because the students did not include any.

Ms. Black says, "We have a list of integers here on the chalkboard. Can someone give an example of a number that is not an integer?"

After several students respond, the list of non-examples for integers includes 0.4, 0.5 , -7.2. Ms. Black says, "Okay, now we have examples of numbers that are integers and numbers that are not integers. Can someone describe what an integer is?"

Joyce confidently says, "It looks like an integer is like a whole number. You know, like not a decimal or fraction."

"Thank you, Joyce that was insightful looking at our examples and non-examples. Does anyone have anything to add to that description?" asks Ms. Black.

Dennis adds to the definition saying, "And some of 'em are negative."

Ms. Black says, "It's hard to sneak anything past you guys. You have a very good eye for this, class. Have you ever considered detective work? (smile) It looks like we have a solid definition now." Ms. Black wraps up by writing a summary on the chalkboard. The summary consists of the input from Joyce and Dennis, using their words for describing integers. Ms. Black writes the following description for integers, "Like whole numbers, not fractions or decimals, and some negative."

Moving the game forward, Ms. Black asks the class, "Do you have enough information to determine your score using the first scoring criteria?" As Ms. Black scans the room she sees mostly bobbing heads and smiles. No one responds negatively, so Ms. Black walks around the room as the students determine their score. As she traverses the room she observes students' work and engages individual students by asking them to explain their thinking about their scoring.

After waiting sufficiently to give students an opportunity to determine their scores, Ms. Black uncovers scoring criteria number two, which reads, "Two points if the product of your numbers is negative." Students score this quickly. Ms. Black can tell that no students included negative numbers in their lists from the mumbling from the students and her casual observations of student papers.

Ms. Black poses the following question to the students, "What would you need to have in your list in order to add two points to your score for this criteria?" Ms. Black and the students discuss this for several minutes. They also engage in a brief discussion about how this criteria might impact the score of the first criteria.

The third criteria is revealed on the overhead and reads, "five points if the sum of the natural numbers is greater than 500." This elicits a discussion of natural numbers similar to the integer discussion. Ms. Black does not provide definitions for any of the number types. She gets the students to create

examples and non-examples of each type of number and records them on the chalkboard and then she gets them to describe the number types using their own words. She will later give students the opportunity to compare their definitions to the formal definitions given in the textbook.

The game proceeds as described above, with the students getting more and more interested as the scoring criteria are revealed. After number five, Mark asks, "Are we going to use these guidelines when we play again?" Ms. Black affirms his question and the students begin to concentrate even more. When all nine guidelines have been revealed and discussed, the students tally their scores and then write down their new 10 favorite numbers, eager to play another round.

The second time through the game, Ms. Black asks students to raise their hands if they are giving themselves points for a certain criteria. If they are, she selects random students to share their numbers. By the end of the second game, there is an example for each one of the nine scoring criteria written on the chalkboard. The game has become competitive at this point, and the students are really competing to get the highest score. They will play one more time to try to beat their previous score.

Materials/Resources Needed:

For the teacher: A list of 10 criteria for the game that match the characteristics or properties of mathematical concepts being targeted in the lesson (e.g., point assignment for negative integers), candy or trinkets that can be used for prizes to encourage participation during the game.

For the students: Paper, pencil, index cards (optional).

Technology Connections:

Use presentation software to generate an on line scenario that can be used to play the Number Game independently. Students or teachers can generate the electronic version of the Number Game to be played by others in groups or individually.

Attachments:
• Commentary on Student Work
• Sample Student Work A
• Sample Student Work B
• Sample Student Work C
• Sample Student Work D
• Sample Student Work E
• Sample Student Work F
• Sample Student Work G
• Sample Student Work H