Indirect Measurement Technique: Using Trigonometric Ratios — Grade Nine

Ohio Standards Connections

Measurement

Benchmark D
Use proportional reasoning and apply indirect measurement techniques, including right triangle trigonometry and properties of similar triangles, to solve problems involving measurements and rates.
(Grades 8 - 10)

Indicator 4
Use scale drawings and right triangle trigonometry to solve problems that include unknown distances and angle measures.
(Grade 9)

Mathematical Processes Standard

Benchmark A
Formulate a problem or mathematical model in response to a specific need or situation, determine information required to solve the problem, choose method for obtaining this information, and set limits for acceptable solution.
(Grades 8 - 10)

Lesson Summary:
Students will find the height of an object that would be difficult or impossible to measure directly. They will construct and use a clinometer to measure the angle of elevation (or depression). Students will create a sketch of the measurement situation using a right triangle, indicate relevant points in the physical situation, and use basic trigonometric ratios to calculate the desired lengths.

Estimated Duration: *90 minutes (excluding pre- and post-assessments).*

Commentary:
In this lesson, students will apply basic right triangle trigonometric ratios in order to determine the height of an object. They will use a homemade clinometer. There are many important concepts embedded in this activity.

First, there is no obvious right triangle present. Students must be able to visualize an appropriate right triangle and determine what locations are its vertices. Second, it is not immediately obvious to many students exactly what angle is being measured by the clinometer and how that angle is related to the angle of elevation to the height of the object. Third, students must understand the function of the weighted string and how it is related to the other "angles" and "sides" present in the experiment. Another aspect of this problem is the ability to create an equation from the problem situation and solve that equation.

Before completing the measurement activity, students should have ample time to experiment with the clinometer to determine what angle they are measuring and what other parts of the triangle are measurable.

Pre-Assessment:
Students should be provided a problem set that enables them to demonstrate their understanding of the basic trigonometric ratios, sine, cosine and tangent. Emphasis should be given to the tangent ratio and distinguishing the difference between the adjacent side and opposite side with respect to a given angle. The Indirect Measurement Pre-Assessment (Blackline Master #1) provides a sample or you may select problems from your own curriculum materials.

Invite students to share their solutions to each of the three questions on the pre-assessment and explain their thinking. Allow students the opportunity to correct any errors presented. Ask appropriate follow-up questions to ensure that students understand the basics of trigonometric ratios.

Teacher Tip:
Walk around the room while students are working and record several of the students' answers for the tan A. When students have completed the pre-assessment, write the collection of answers you recorded on the board and challenge students to determine which is correct. Have students provide justification for the correct solution. Then, engage students to determine what

Other Related Ohio Standards

Measurement

Benchmark C
Apply indirect measurement techniques, tools and formulas, as appropriate, to find perimeter, circumference and area of circles, triangles, quadrilaterals and composite shapes, and to find volume of prisms, cylinders, and pyramids.
(Grades 8 - 10)

Geometry and Spatial Sense

Benchmark H
Establish the validity of conjectures about geometric objects, their properties and relationships by counter-example, inductive and deductive reasoning, and critiquing arguments made by others.
(Grades 8 - 10)

Benchmark I
Use right triangle trigonometric relationships to determine lengths and angle measures.
(Grades 8 - 10)

might have led to the incorrect answers. Continue this activity until students seem to understand how to select the appropriate sides to find tan A.

During informal observation of student work, look for students having difficulty with sine or cosine. If needed, discuss the other ratios using a process similar to that described above.

Scoring Guidelines:
Informally evaluate students' strengths and weaknesses. Walk around the room and observe the progress and take notes as needed.

Instructional Tip:
This assessment was designed to be an instructional review. All or most students struggle with a specific part of the pre-assessment may indicate the need for a mini-lesson focused on that specific topic prior to continuing with this lesson. Some of the more prevalent errors students encounter includes the following:

- Not remembering the side relationships for each ratio.
- Not realizing that the side designations are related to a specific angle (Is this always challenging or only for angles in certain positions or when triangles are rotated in a different orientation? Note the orientation of the triangle used on the pre-assessment.)
- Not remembering which side is opposite or adjacent (i.e., unfamiliar terminology can be hard to recall.)
- Difficulty performing necessary computations or algebraic simplification.

Post-Assessment:
Students will describe another situation where this technique could be used to measure an object from their own environment. Then, they will use the clinometer and this technique to determine the size (height, length, etc.) of the object chosen. The object they select should not be something measured during this lesson.

Students should provide a written explanation of how they set up the situation to apply the technique. Include explanations for finding relevant angles (angle of elevation or depression) and measuring necessary lengths. Students should support their description and explanation with labeled sketches to show the situation including all pertinent objects, measures and variables.

Scoring Guidelines:
The post-assessment will be scored using a rubric. Students should be provided with a copy of the rubric prior to the assessment. Make sure students understand the assignment as well as all aspects of the scoring rubric prior to beginning the assessment.

Teacher Tip:
It is suggested that you set aside time with your students to discuss the parameters of the assessment prior to assigning it. You may want to consider

making adjustments to the assignment or rubric based upon input from the students. The last column of the rubric was left blank intentionally to encourage student–teacher negotiated additions.

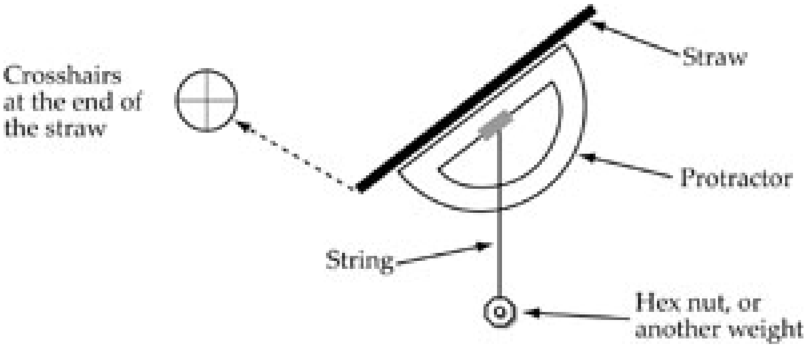
Sample Analytic Scoring Rubric:

Score	Situation description	Explanation of setup and execution	Labeled sketch	Other criteria
4	Clearly stated and accurate	Understanding of measurement technique evident	Sketch with complete labeling and scaled reasonably accurate	
3	Clearly stated, but not very reasonable	One or more minor concepts missing or not mentioned	Sketch with most things labeled and scaled adequately	
2	Clearly stated, but copied from class or HW	One key concept missing	Sketch with missing labels and poorly scaled	
1	Not clearly stated	An attempt is made in written format	Sketch attempted, but hinders correct solution	
0	Not included or copied from class or HW	No evidence demonstrated or no attempt	Not included or copied from class or HW	

Instructional Procedures:

1. Prior to the lesson, select an object to be measured. Some possibilities include the height of a flagpole or the school building, the height of a basketball backboard or a marked height on a wall like the top of a window in your classroom. The object should be tall enough to make it difficult to measure directly.
2. Create one clinometer in advance using the directions and the diagram provided below:
   - Tape one end of a 6-inch string to the midpoint (directly between the 0^o

and the 180^{o} mark on the protractor) on the straight side of the protractor.
- Tape two pieces of string across the diameter on one end of the straw forming a crosshair.
- Tape the straw to the straight side of the protractor.
- Tie the hex nut (or weight) to the other end of the string.



3. Inform students that a clinometer is a measuring device and show them the pre-made model. Demonstrate looking through the end of the straw (without the crosshair) and getting the angle measurement (wait for the string to stop swaying and then pinch the string against the curved edge of the protractor).

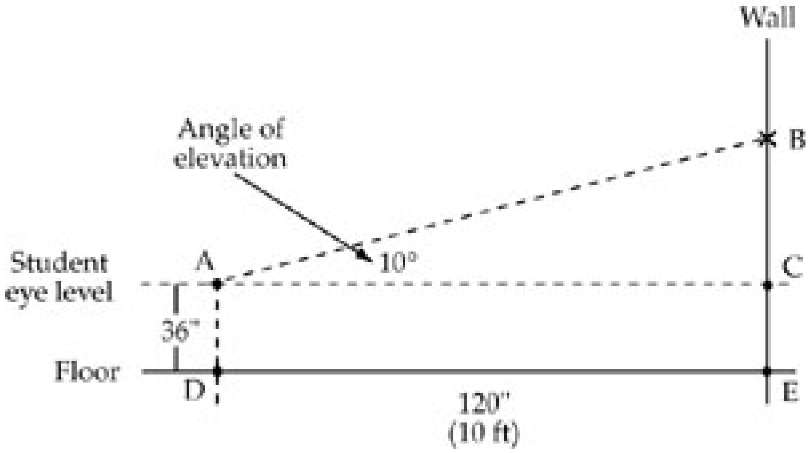
4. Distribute Using a Clinometer (Blackline Master #2) with the directions for building a clinometer. Organize students into pairs and instruct them to use the directions and the pre-made model to build a clinometer. Students should work in pairs to practice sighting objects either above or below their line of sight.

5. Encourage students to experiment with the clinometers, by asking them questions similar to the following:
   - What angle does the clinometer measure with respect to what you are looking at through the sight?
   - How would you define "angle of elevation" or "angle of depression" as it relates to the objects you are sighting?
   - How is the angle of elevation (depression) related to the angle measurement where the string crosses the scale of the protractor?
   - Identify a right triangle that includes the object and the angle of elevation (depression)? What are the other parts (sides/angles) of that triangle?
   - What parts of your triangle could you actually measure? How could those parts be used to determine the measurements you want to know?

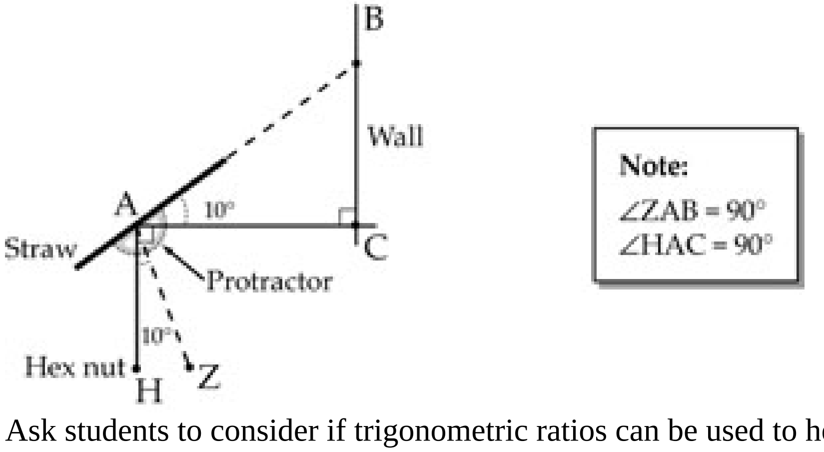
6. Facilitate a whole class discussion to share observations and to review answers to the questions used for the clinometer exploration.

<u>Instructional Tip:</u>
The next step describes a whole class demonstration. Some students may be ready to work in groups independently with a little guidance. Determine the best strategy for your students and their understanding.

7. Inform students they will help you to measure an object (point out or describe whatever you pre-selected to measure) using a clinometer. For example, if you selected to measure the distance from the floor to an "X" on a wall the following would apply:
   - Place an X on the wall with masking tape (for a whole class demonstration, this should be done prior to class).
   - Select a student to sit in a chair 10 feet or 120 inches from the wall (measure from the wall to the student's eyes).
   - The student will use the clinometer to measure the angle of elevation by aligning the crosshair with the center of the "X" on the wall.
   - Measure the height from the floor to the student's eyes. Ask students why this measurement is needed. (Note: This measurement will be added to the solution of the trigonometry problem.)
   - Ask the students to label the diagram of the situation using the sketch provided on Blackline Master #2 and shown below. Students may be encouraged to work with a partner for this portion, as needed.



   - After waiting sufficiently for students to think about the situation, select a student to share his/her labeled sketch. Discuss as a class until a consensus is reached and a correctly labeled sketch is completed.
   - Include in your discussion how to find the angle of elevation. Refer to the diagram below for assistance. Ask students about the different 90° angles and to explain why $\angle BAC \cong \angle HAZ$.



   - Ask students to consider if trigonometric ratios can be used to help find BE (distance from the point on the wall to the floor in the problem

situation.) If so, which ratio and how? If not, why not?
- Provide sufficient time for students to consider the problem and then discuss the solution as a class. If all goes well, students should provide most of the input during the discussion.

Solution for Class Demonstration:

AC was measured as 10 feet (120 inches) from the student's eyes to the wall and AD was measured (we are using 36^o for the sample). BAC was found using the clinometer and should be about 10^o. BE can be found as follows:
$\tan 10^o = BC/120$

$120 \tan 10^o = BC$

BC = 21 inches

then to get BE:

BE = BC + AD

BE = 21 + 36

BE = 57 inches

BE = 4' 9"

8. Organize the students into groups of two or three and provide additional practice using this technique. They can measure things around the school building or outside. Challenge the students to find situations to measure that would involve an angle of depression (i.e., something down stairs or down hill) or to measure something that might involve a different trigonometric ratio (i.e., you can measure the length of the hill but cannot measure an object directly in front of you). Be prepared to offer suggestions for measuring situations.

Teacher Tip:
Optionally provide students with a list of measurement situations from around the school grounds. Then allow students to select from the list. This may be useful to keep students engaged and on task.

9. Discuss objects measured, strategies used for measuring, challenges encountered, and solutions.

Differentiated Instructional Support:
Instruction is differentiated according to learner needs, to help all learners either meet the intent of the specified indicator(s) or, if the indicator is already met, to advance beyond the specified indicator(s).
- Students may work in collaborative groups to complete this lesson. It may be beneficial to have heterogeneous groups to enable peer tutoring during the activities.

- Provide students with pre-drawn diagrams and let the students record the measures (refer to Blackline Master #2).
- Consider using the Indirect Measurement Techniques – Grade Eight lesson with students who are not ready to work with the trigonometric ratios.
- Challenge the students to use the clinometer to measure objects involving an angle of depression instead of an angle of elevation; e.g., measure something that is situated below eye level.
- Challenge the students to use the clinometer to measure objects involving the other trigonometric ratios, sine or cosine; e.g., if there were a tree situated at a bottom of a hill, then a student could measure the length of the hill using a tape measure and walking the hill. Then use the cosine to find the distance from a person standing on the top of the hill to the top of the tree or use the sine to find the height of the tree.

Extension:

These are ideas for all students to continue learning on this topic - in the classroom or outside of the classroom.

- Select three indirect measurement situations for the students to use their clinometers outside the school that would be difficult to measure directly. Each measurement situation will be a station.
  - Divide the class into three groups (Groups 1, 2 & 3).
  - Each group will be given time to collect data (i.e., agree on the angle measure using their clinometers), measure the distance between the clinometer and the ground and to find any other measures needed.
  - Then, groups should discuss set-up (i.e., whether to use an angle of elevation or depression and which trigonometric ratio should be used to find the desired measurement).
  - Groups will rotate through the three stations and collect data and discuss set-up at each station.
  - Complete the calculations and create sketches outside of class (optional). Each student should create a scale drawing and show the work needed to calculate the specified measure for each station.
  - Discuss solutions for each station within the three groups in class. The larger groups need to reach consensus, if possible, or at a minimum agree on a final answer.
  - Prepare for students to do a jigsaw discussion about how their groups performed the indirect measurements at each of the three stations and to share their group's findings:
    a. Organize the students into groups of three students with at least one student from each group.
    b. The students will take turns discussing the process or steps their group used to find the measures at each station.
  - Students should discuss challenges and how their group worked through them.
  - Students should reconcile differences in their solutions. For example, each group probably stood at different distances from the object, but the answers should be the same or close. Why are the answers not different? They should conclude that the distance from the object does not affect the final solution.
  - Circulate the room and provide assistance as needed and keep the

student discussions on task and moving so they can progress through to consider objects 2 and 3. Refrain from providing the students with the right answers.

Homework Options and Home Connections:

- Each student must choose an object near his/her home and find the height using the clinometer. A written description of the situation scale drawing and work must accompany the solution.
- Resources for parent and family involvement are available on the Internet.

Interdisciplinary Connections:

- Environmental and Agricultural Sciences - This indirect measurement technique could be used to find the height of trees. The solution could be used when determining the amount of lumber or firewood a tree would likely produce.
- Geography - This indirect measurement technique could be used to find the height of natural landmarks such as mountains and caverns; e.g., cartographers use similar techniques when making maps.

Key Vocabulary:

- clinometer
- basic trigonometric ratios
- tangent (tan)
- sine (sin)
- cosine (cos)
- indirect measurement
- angle of elevation
- angle of depression

Technology Connections:

- A calculator with trigonometric functions will make the lesson more manageable and eliminate the need for looking up values in tables. Interactive calculators with trigonometric functions are available on the Internet.
- Students can use word processing and productivity software to report their findings visually and in writing.
- Students could use a database to input their data for the class to view and compare results. Interactive charts could also be created from the collected data.

Materials/Resources Needed:

For the teacher: Blackline Master #1 (Indirect Measurement Pre-Assessment), Blackline Master #2 (Indirect Measurement Using A Clinometer), a pre-assembled clinometer to show to students, parts needed to assemble a clinometer (transparent tape, a small spool of thread, drinking straw, protractor, string, hex nut or weighting device), tape measure, masking tape.

For the students: Parts needed to assemble a clinometer (transparent tape, thread, drinking straw, protractor, string, hex nut), tape measure, masking tape.

Attachments:
- Indirect Measurement Pre-Assessment (Blackline Master #1)
- Using a Clinometer (Blackline Master #2)